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Time Truncated Two Stage Group Sampling Plan For Various Distributions

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Abstract: **In this paper, two stage group sampling plan is developed for a truncated life test when the life time of an item follows different distributions. The parameters of the proposed plan such as minimum number of testers and probability of lot acceptance are determined when the consumer's risk and test termination time are specified. The results are discussed with the help of tables and examples.**

Keywords: **Truncated life test, Generalized Exponential distribution, Marshall–Olkin extended Lomax distribution, Marshall – Olkin extended exponential distribution, Weibull Distribution, Generalised Rayleigh Distribution, Inverse Rayleigh Distribution, Operating characteristics, Consumer's risk.**

1. INTRODUCTION

Acceptance sampling plan is an inspecting procedure in statistical quality control or reliability tests, widely used in the field of industry and business to manage the product reliability and to make the decision of accepting or rejecting the product by the consumer .Today it is very difficult to observe the complete life time of the product that are designed with high reliability**.** The acceptance sampling plans for a truncated life test are frequently used to determine the sample size from a lot under consideration. For a usual sampling plan it is assumed that only a single item is put in a tester. However, in practice more than one tester can be used to test the multiple numbers of items at a time because testing time and cost can be saved by testing items simultaneously. For this type of testers the number of items to be equipped in a tester is given by the specification. The acceptance sampling plan under this type of testers will be called a group acceptance sampling plan. The items in a tester are referred as a group and the number of items in a group is called the group size. This type of testers is frequently used in sudden death testing. The sudden death tests are discussed by Pascual and Meeker (1998) and Vlcek et al (2003). In a time truncated acceptance sampling plan, a random sample is chosen from a submitted lot of items and put them on test where the number of failures is observed until the pre-specified time. If the number of failures is greater than the specified acceptance number, then the submitted lot will be rejected. An ordinary time truncated acceptance sampling plan have been discussed by many authors, Goode and Kao (1961), Gupta and Groll(1961), Baklizi and EI Masri(2004), Rosaiah and Kantam (2005) and Tsai, Tzong and Shou(2006), Balakrishnan, Victor Leiva & Lopez (2007). Radhakrishnan and Alagirisamy was the first to attempt the attribute group acceptance sampling plan using weighted binomial distribution for Pareto distribution to determine the minimum number of groups. Sudamani ramaswamy and Priyah anburajan, (2012), discussed the two stage group acceptance sampling plans on truncated life tests for marshall – olkin extended distributions. In this paper, an approach of designing two stage group acceptance sampling plan for truncated life test is proposed, when the lifetime of the items follows different life time distributions. The distributions considered in this paper are Marshall – Olkin extended Lomax distribution, Marshall – Olkin extended exponential distribution, Weibull distribution, Generalised exponential distribution, Generalised Rayleigh distribution, Inverse Rayleigh distribution. The test termination time and mean ratios are specified. The minimum number of testers is obtained such that it satisfies the consumer's risk. The probability of acceptance is also determined. The tables of the design parameter are provided for easy selection of the plan. The results are analysed with the help of tables and examples.

2. GLOSSARY OF SYMBOLS

n	-	Size of the sample
d	-	Number of defectives
c_1	-	Acceptance number of the first sample
c_2	-	Acceptance number of the second sample
$P_a(p)$	-	Probability of acceptance of a lot submitted for inspection
α	-	Producer's risk
β	-	Consumer's risk
σ	-	Scale parameter
T	-	Prefixed time
μ	-	Mean life
μ_0	-	Specified life
p	-	Failure probability
m, λ, γ	-	Shape parameter
a	-	Test termination time multiplier
g_1	-	Number of groups in first stage
g_2	-	Number of groups in second stage
r	-	Number of items in a group.

3. DISTRIBUTIONS

The following are the distributions used in this paper:

(i) Generalized exponential distribution:

The cumulative distribution function (cdf) of the generalized exponential distribution is given by

$$F(t,\sigma) = \left(1 - e^{-\frac{t}{\sigma}}\right)^{\lambda} \quad , \ t > 0, \sigma > 0 \qquad \textbf{(1)}$$

Where σ is a scale parameter and λ is the shape parameter and it is fixed as 2.

(ii) Marshall – Olkin extended Lomax distribution:

The cumulative distribution function (cdf) of the Marshall – Olkin extended Lomax distribution is given by

$$F(t,\sigma) = \frac{(1+t/\sigma)^{\theta} - 1}{(1+t/\sigma)^{\theta} - \bar{\gamma}}, \bar{\gamma} = 1 - \gamma \quad , \ t > 0, \sigma > 0 \qquad \textbf{(2)}$$

Where σ is a scale parameter and θ and γ are the shape parameters and they are fixed as 2.

(iii) Marshall – Olkin extended exponential distribution:

The cumulative distribution function (cdf) of the Marshall – Olkin extended exponential distribution is given by

$$F(t,\sigma)=\frac{1-e^{-t/\sigma}}{1-\bar{\gamma}e^{-t/\sigma}},\ \bar{\gamma}=1-\gamma \quad ,\ t>0,\ \sigma>0 \tag{3}$$

Where σ is a scale parameter and γ is the shape parameter and it is fixed as 2.

(iv) Weibull distribution:

The cumulative distribution function (cdf) of the Weibull distribution is given by

$$F(t,\sigma)=1-e^{-\left(\frac{t}{\sigma}\right)^m} \quad ,\ t>0,\ \sigma>0 \tag{5}$$

Where σ is a scale parameter.

(v) Generalised *Rayleigh distribution*

(vi) The cumulative distribution function (cdf) of the Rayleigh distribution is given by

$$F(t,\sigma)=\left(1-e^{-\frac{t^2}{2\sigma^2}}\right) \quad ,\ t>0,\ \sigma>0 \tag{4}$$

(vii) where σ is a scale parameter ***Inverse Rayleigh distribution***

The cumulative distribution function (cdf) of the Inverse Rayleigh distribution is given by

$$F(t,\sigma)=e^{-\frac{\sigma^2}{t^2}} \quad ,\ t>0,\ \sigma>0 \tag{6}$$

Where σ is a scale parameter.

If some other parameters are involved, then they are assumed to be known. The failure probability of an item by time t_0 is given by

$$p = F(t_{0\,:}\,\sigma) \tag{7}$$

The quality of an item is usually represented by its true mean lifetime. Let us assume that the true mean μ can be represented by the scale parameter. In addition, it is convenient to specify the test time as a multiple of the specified life so that $a\mu_0$ and the quality of an item as a ratio of the true mean to the specified life (μ/μ_0).

Then we can rewrite (6) as a function of 'a' (termination time) and the ratio μ/μ_0.

$$p = F(a\,\mu_0 : \mu/\mu_0) \tag{8}$$

Here when the underlying distribution is the inverse Rayleigh distribution

$$p=\exp\left(-\frac{1}{a^2\pi}\left(\frac{\mu}{\mu_0}\right)^2\right) \tag{9}$$

When the underlying distribution is Generalised Rayleigh distribution

$$p = 1 - \sum_{j=0}^{k} \left(\frac{\left(\frac{am}{\mu / \mu_0} \right)^{2j} e^{-\left(\frac{am}{\mu / \mu_0} \right)^2}}{j!} \right) \tag{10}$$

Where m = Γ(k+1/2)/ Γ(k+1)

When the underlying distribution is the Marshall – Olkin extended exponential distribution

$$p = \frac{1 - e^{-\frac{1.5708a}{\mu / \mu_0}}}{1 - \bar{\gamma} e^{-\frac{1.5708a}{\mu / \mu_0}}}, \bar{\gamma} = 1 - \gamma \tag{11}$$

When the underlying distribution is the Marshall – Olkin extended Lomax distribution

$$p = \frac{\left[1 + 1.5708a / (\mu / \mu_0)\right]^{\theta} - 1}{\left[1 + 1.5708a / (\mu / \mu_0)\right]^{\theta} - \bar{\gamma}}, \bar{\gamma} = 1 - \gamma \tag{12}$$

When the underlying distribution is the Generalised exponential distribution

$$p = 1 - e^{-\frac{1.2279a}{\mu / \mu_0}} \tag{13}$$

When the underlying distribution is the Weibull distribution

$$p = 1 - e^{-\left(\frac{ba}{\mu / \mu_0} \right)^m} \tag{14}$$

4. OPERATING PROCEDURE FOR TWO – STAGE GROUP ACCEPTANCE SAMPLING PLAN FOR TRUNCATED LIFE TEST

According to Abbur Razzaque Mughal, Muhammad Hanif, Azhar Ali Imran, Muhammad Rafi and Munir Ahmad [3] for the two – stage group acceptance sampling plan for truncated life test.

1. (First stage) Draw the first random sample size n_1 from a lot, allocate r items to each of g_1 groups (or testers) so that $n_1 = rg_1$ and put them on test for the duration of t_0. Accept the lot if the number of failures from each group is c_1 or less. Truncate the test and reject the lot as soon as the number of failures in any group is larger than c_2 before t_0. Otherwise, go to the second stage.
2. (Second stage) Draw the second random sample of size n_2 from a lot, allocate r items to each of g_2 groups so that $n_2 = rg_2$ and put them on test for t_0. Accept the lot if the number of failures in each group is c_1 or less. Truncate the test and reject the lot if the number of failures in any group is larger than c_1 before t_0.

The following is the operating procedure for two – stage group acceptance sampling plan for life test in the form of a flow chart.

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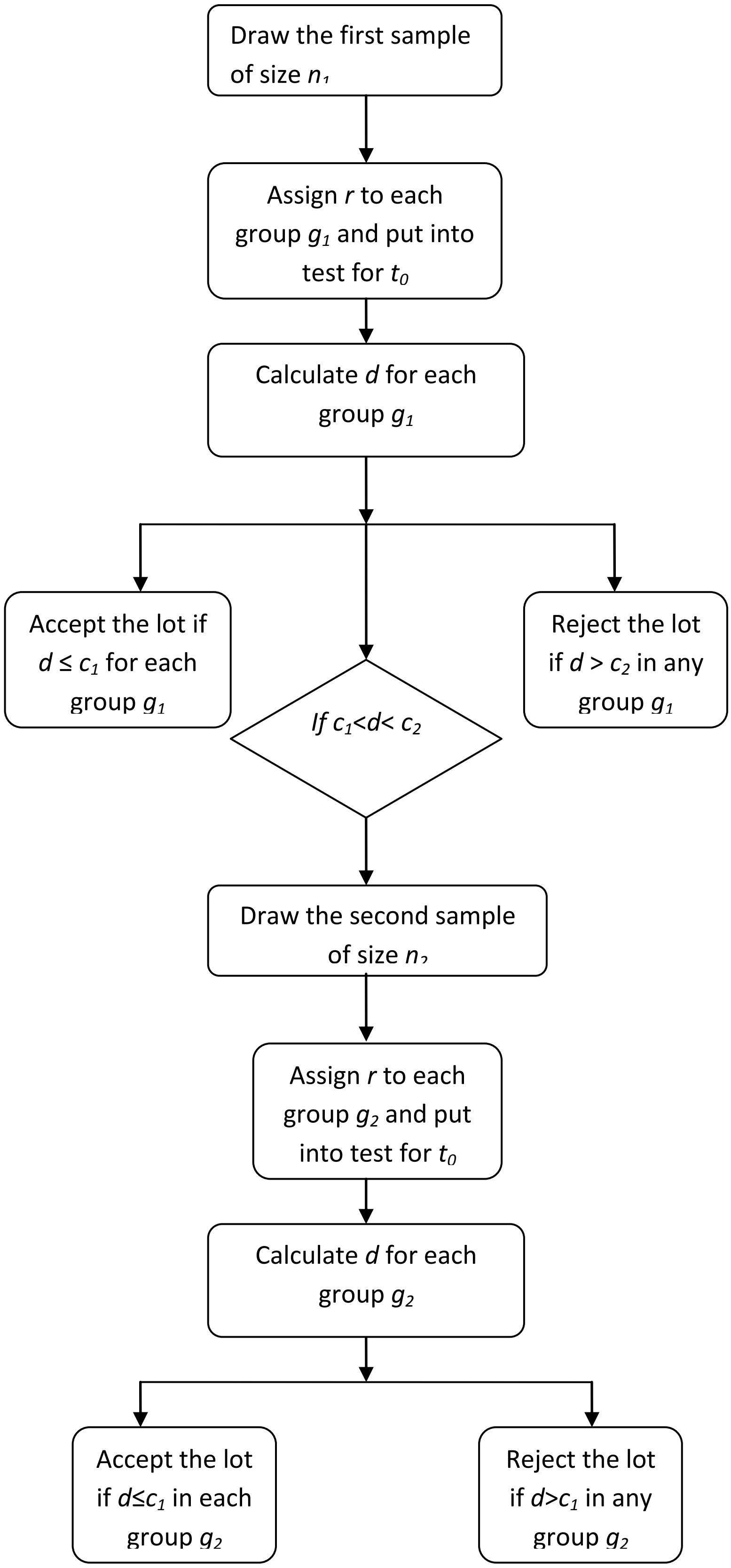
5. FLOW CHART



The two – stage group acceptance sampling plan constitute the design parameters of g_1, g_2, c_1 and c_2 when the number of testers r are specified.

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6. CONSTRUCTION OF TABLES

The probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (oc) function of the sampling plan. Once the minimum number of testers is obtained one may be interested to find the probability of acceptance of a lot when the quality of the product is good enough.

The probability of lot acceptance at the first stage can be evaluated as,

$$P_a^{\,1} = \left[\sum_{i=0}^{c_1} \binom{r}{i} p^i (1-p)^{r-i} \right]^{g_1} \tag{15}$$

The probability of lot rejection at the first stage is given by,

$$P_r^{\,1} = 1 - \left[\sum_{i=0}^{c_2} \binom{r}{i} p^i (1-p)^{r-i} \right]^{g_1} \tag{16}$$

where the probability of lot acceptance at the second stage is

$$P_a^{\,2} = \left[1 - (P_a^{\,1} + P_r^{\,1})\right] \left[\sum_{i=0}^{c_1} \binom{r}{i} p^i (1-p)^{r-i} \right]^{g_2} \tag{17}$$

Therefore, the probability of lot acceptance for the proposed two – stage group acceptance sampling plan is given by

$$L(p) = P_a^{\,1} + P_a^{\,2} \tag{18}$$

$$1 \le g_2 \le g_1 \,, \; 0 \le c_1 \le c_2$$

The minimum number of testers required can be determined by considering the consumer's risk when the true mean life equals the specified mean life ($\mu = \mu_0$) (worst case) by means of the following inequality:

$$L(p_0) \le \beta \tag{19}$$

Where p_0 is the failure probability at $\mu = \mu_0$. Here minimum number of testers r is obtained using (18) and (19).The failure probabilities are obtained by fixing the time multiplier 'a' as 0.7, 0.8, 1.0, 1.2, 1.5 and 2.0 and the mean ratios μ/μ_0 as 2, 4, 6, 8, 10 and 12. The minimum number of testers is determined by fixing the number of groups and the consumer's risk β as 0.25, 0.10, 0.05, and 0.01 in the equations (18) and (19). The minimum numbers of testers is determined for the above cited distributions and are presented in the Table 1 to Table 6 respectively. The minimum sample size for the above distributions can be obtained by using $n = rg$, from Table 1 to Table 6. The tables indicates that, as the test termination time multiplier 'a' increases, the number of testers 'r' decreases, i.e., a smaller number of testers is needed, if the test termination time multiplier increases for a fixed number of groups. The probability of acceptance are also calculated and are presented in the Table 7 to Table 12 when the life time of the item follows different distributions.

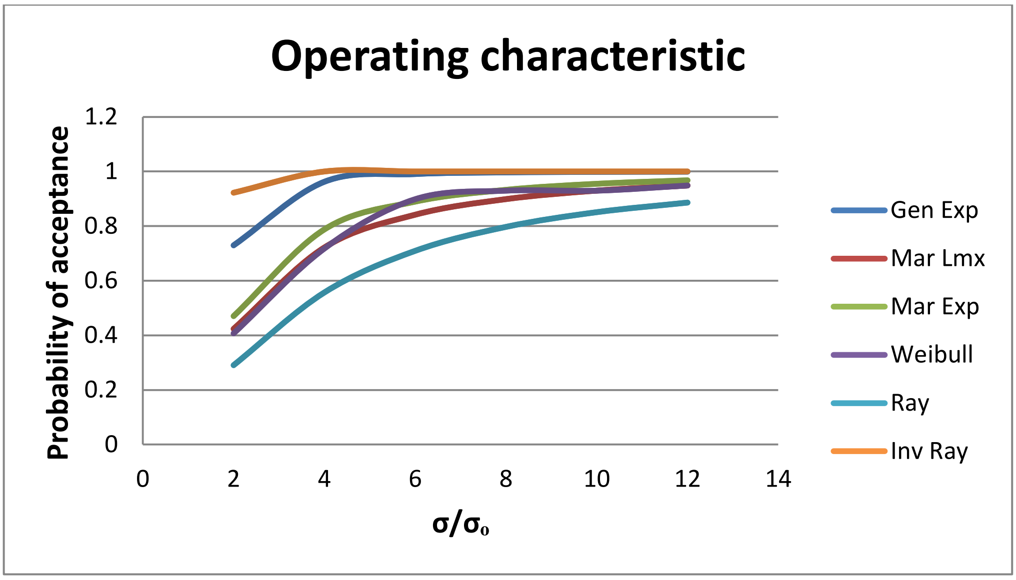

Figure 1: OC curve for Probability of acceptance against μ/μ_0 of two stage group sampling plan when the life time of the item follows different distributions.

Table1: Minimum number of testers for the two- stage group sampling plan with $c_1 = 0$ and $c_2 = 2$ when the life time of the item follows Generalized exponential distribution

β	g1	g2	a					
			0.7	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	**2**	**1**	4	3	3	2	2	2
	3	**1**	4	3	3	2	2	2
	2	**2**	3	3	2	2	2	2
	3	**2**	**3**	**2**	**2**	**2**	**2**	**2**
0.10	**2**	**1**	5	5	4	3	2	2
	3	**1**	5	4	3	3	2	2
	2	**2**	4	3	2	2	2	2
	3	**2**	**4**	**3**	**2**	**2**	**2**	**2**
0.05	**2**	**1**	6	5	4	3	3	2
	3	**1**	6	5	4	3	3	2
	2	**2**	5	4	3	3	2	2
	3	**2**	**4**	**4**	**3**	**2**	**2**	**2**
0.01	**2**	**1**	9	7	5	4	4	3
	3	**1**	8	6	5	4	3	3
	2	**2**	7	6	4	3	3	2
	3	**2**	**5**	**5**	**4**	**3**	**2**	**2**

Table 2: Minimum number of testers for the two- stage group sampling plan with $c_1 = 0$ and $c_2 = 2$ when the life time of the item follows Marshall – Olkin extended Lomax distribution

β	g1	g2	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	1	2	2	2	2	2	2
	3	1	2	2	2	2	2	2
	2	2	2	2	2	2	2	2
	3	2	2	2	2	2	2	2
0.10	2	1	3	3	2	2	2	2
	3	1	3	3	2	2	2	2
	2	2	2	2	2	2	2	2
	3	2	2	2	2	2	2	2
0.05	2	1	3	3	3	2	2	2
	3	1	3	3	3	2	2	2
	2	2	2	2	2	2	2	2
	3	2	2	2	2	2	2	2
0.01	2	1	4	4	3	3	3	3
	3	1	4	4	3	3	3	3
	2	2	3	3	2	2	2	2
	3	2	3	3	2	2	2	2

Table 3: Minimum number of testers for the two- stage group sampling plan with $c_1 = 0$ and $c_2 = 2$ when the life time of the item follows Marshall – Olkin extended exponential distribution

β	g_1	g_2	a					
			0.7	0.8	1	1.2	1.5	2
0.25	2	1	3	2	2	2	2	2
	3	1	3	2	2	2	2	2
	2	2	2	2	2	2	2	2
	3	2	2	2	2	2	2	2
0.10	2	1	4	3	3	2	2	2
	3	1	3	3	3	2	2	2
	2	2	3	2	2	2	2	2
	3	2	2	2	2	2	2	2
0.05	2	1	4	4	3	3	2	2
	3	1	4	3	3	3	2	2
	2	2	3	3	2	2	2	2
	3	2	3	2	2	2	2	2
0.01	2	1	5	5	4	3	3	2
	3	1	5	4	4	3	3	2
	2	2	4	4	3	2	2	2
	3	2	4	3	3	2	2	2

Table 4: Minimum number of testers for the two- stage group sampling plan with $c_1 = 0$ and $c_2 = 2$ when the life time of the item follows Weibull distribution

β	g1	g2	a: 0.7	0.8	1	1.2	1.5	2.0
	2	1	2	2	2	2	2	2
	3	1	2	2	2	2	2	2
	2	2	2	2	2	2	2	2
0.25	3	2	2	2	2	2	2	2
	2	1	3	2	2	2	2	2
	3	1	3	2	2	2	2	2
	2	2	2	2	2	2	2	2
0.10	3	2	2	2	2	2	2	2
	2	1	3	3	2	2	2	2
	3	1	3	3	2	2	2	2
	2	2	2	2	2	2	2	2
0.05	3	2	2	2	2	2	2	2
	2	1	4	3	3	3	2	2
	3	1	4	3	3	3	2	2
	2	2	3	3	2	2	2	2
0.01	3	2	3	2	2	2	2	2

Table 5: Minimum number of testers for the two- stage group sampling plan with $c_1 = 0$ and $c_2 = 2$ when the life time of the item follows Generalised Rayleigh distribution

β	g_1	g_2	a: 0.7	0.8	1	1.2	1.5	2.0
	2	1	3	3	2	2	2	2
	3	1	2	2	2	2	2	2
	2	2	2	2	2	2	2	2
0.25	3	2	2	2	2	2	2	2
	2	1	4	3	3	3	2	2
	3	1	3	3	2	2	2	2
	2	2	2	2	2	2	2	2
0.10	3	2	2	2	2	2	2	2
	2	1	4	4	4	3	3	2
	3	1	3	3	3	2	2	2
	2	2	2	2	2	2	2	2
0.05	3	2	2	2	2	2	2	2
	2	1	6	5	5	4	3	3
	3	1	4	4	3	3	3	3
	2	2	3	3	2	2	2	2
0.01	3	2	3	3	2	2	2	2

Table 6: Minimum number of testers for the two- stage group sampling plan with $c_1 = 0$ and $c_2 = 2$ when the life time of the item follows Inverse Rayleigh distribution

β	g_1	g_2	a					
			0.7	0.8	1	1.2	1.5	2.0
0.25	2	1	3	2	2	2	2	2
	3	1	2	2	2	2	2	2
	2	2	2	2	2	2	2	2
	3	2	2	2	2	2	2	2
0.10	2	1	3	3	2	2	2	2
	3	1	3	3	2	2	2	2
	2	2	3	2	2	2	2	2
	3	2	2	2	2	2	2	2
0.05	2	1	4	3	3	2	2	2
	3	1	4	3	3	2	2	2
	2	2	3	2	2	2	2	2
	3	2	3	2	2	2	2	2
0.01	2	1	5	4	3	3	3	2
	3	1	5	4	3	3	3	2
	2	2	4	3	3	2	2	2
	3	2	3	3	2	2	2	2

Table 7: Probability of acceptance for the two- stage group sampling plan with $g_1 = 2$ & $g_2 = 1$, when the life time of the item follows Generalized exponential distribution

β	r	a	μ/μ_0					
			2	4	6	8	10	12
0.25	4	0.7	0.729612	0.9625017	0.990667308	0.996719226	0.998574088	0.99928587
	3	0.8	0.754029	0.965531	0.991333	0.996933	0.998661	0.999327
	3	1.0	0.605988	0.930611	0.981196	0.993111	0.996933	0.99844
	2	1.2	0.662388	0.94048	0.983651	0.993948	0.997285	0.998611
	2	1.5	0.504914	0.887525	0.966078	0.986857	0.993948	0.996851
	2	2.0	0.296901	0.769678	0.91899	0.966078	0.983651	0.991239
0.10	5	0.7	0.636278	0.9442136	0.985759571	0.994944555	0.997792219	0.998891321
	5	0.8	0.527692	0.915476	0.977348	0.991781	0.996367	0.998162
	4	1.0	0.456816	0.887443	0.968068	0.988082	0.994645	0.997262
	3	1.2	0.462364	0.882228	0.965531	0.986904	0.994049	0.996933
	2	1.5	0.504914	0.887525	0.966078	0.986857	0.993948	0.996851
	2	2.0	0.296901	0.769678	0.91899	0.966078	0.983651	0.991239
0.05	6	0.7	0.547007	0.923465	0.979970633	0.992819997	0.996849466	0.998413676
	5	0.8	0.527692	0.915476	0.968345	0.988372	0.99483	0.997375
	4	1.0	0.456816	0.887443	0.968068	0.988082	0.994645	0.997262
	3	1.2	0.462364	0.882228	0.965531	0.986904	0.994049	0.996933
	3	1.5	0.285191	0.78902	0.930611	0.972098	0.986904	0.993111
	2	2.0	0.296901	0.769678	0.91899	0.966078	0.983651	0.991239
0.01	9	0.7	0.324807	0.8506386	0.957970199	0.984495154	0.993100079	0.996498182
	7	0.8	0.342495	0.853183	0.958176	0.98445	0.993044	0.996457
	5	1.0	0.332664	0.839164	0.952304	0.981873	0.991781	0.995775
	4	1.2	0.303253	0.81482	0.942461	0.977585	0.989684	0.994645
	4	1.5	0.145654	0.684181	0.887443	0.953113	0.977585	0.988082
	3	2.0	0.109392	0.605988	0.843404	0.930611	0.965531	0.981196

Table 8: Probability of acceptance for the two- stage group sampling plan with $g_1 = 2$ & $g_2 = 1$when the life time of the item follows Marshall – Olkin extended Lomax distribution

	r	a	μ/μ_0					
			2	4	6	8	10	12
0.25	2	0.7	0.423829	0.7216867	0.841742728	0.898820186	0.92997169	0.948733903
	2	0.8	0.363013	0.671577	0.807607	0.874951	0.912547	0.93552
	2	1.0	0.268257	0.577977	0.73873	0.824752	0.874951	0.906509
	2	1.2	0.200975	0.495243	0.671577	0.773118	0.834969	0.874951
	2	1.5	0.134146	0.392166	0.577977	0.696413	0.773118	0.824752
	2	2.0	0.073532	0.268257	0.446406	0.577977	0.671577	0.73873
0.10	3	0.7	0.208675	0.539287	0.714359	0.808230	0.863058	0.897552
	3	0.8	0.157893	0.473902	0.661725	0.768016	0.832043	0.873134
	2	1.0	0.268257	0.577977	0.738730	0.824752	0.874951	0.906509
	2	1.2	0.200975	0.495243	0.671577	0.773118	0.834969	0.874951
	2	1.5	0.134146	0.392166	0.577977	0.696413	0.773118	0.824752
	2	2.0	0.073532	0.268257	0.446406	0.577977	0.671577	0.73873
0.05	3	0.7	0.208675	0.539288	0.714359	0.808230	0.863058	0.897552
	3	0.8	0.157893	0.473902	0.661725	0.768016	0.832043	0.873134
	3	1.0	0.090664	0.362647	0.562504	0.687856	0.768016	0.821504
	2	1.2	0.200975	0.495243	0.671577	0.773118	0.834969	0.874951
	2	1.5	0.134146	0.392166	0.577977	0.696413	0.773118	0.824752
	2	2.0	0.073532	0.268257	0.446406	0.577977	0.671577	0.73873
0.01	4	0.7	0.090301	0.382788	0.587049	0.710211	0.787022	0.837422
	4	0.8	0.058742	0.314777	0.522246	0.656218	0.743100	0.801568
	3	1.0	0.090664	0.362647	0.562504	0.687856	0.768016	0.821504
	3	1.2	0.052551	0.275543	0.473902	0.610929	0.703720	0.768016
	3	1.5	0.023775	0.181517	0.362647	0.505782	0.610929	0.687856
	3	2.0	0.006849	0.090664	0.228979	0.362647	0.473902	0.562504

Table 9: Probability of acceptance for the two- stage group sampling plan with $g_1 = 2$ & $g_2 = 1$when the life time of the item follows Marshall – Olkin extended exponential distribution

β	r	a	μ/μ_0					
			2	4	6	8	10	12
	3	0.7	0.470391	0.788016	0.889612	0.932799	0.954908	0.967685
	2	0.8	0.605021	0.858607	0.929495	0.958047	0.972242	0.980295
	2	1	0.485785	0.797724	0.895954	0.93715	0.958047	0.970046
	2	1.2	0.382298	0.733647	0.858607	0.913271	0.941585	0.958047
	2	1.5	0.259878	0.636675	0.797724	0.872988	0.913271	0.93715
0.25	2	2	0.130851	0.485785	0.690372	0.797724	0.858607	0.895954
	4	0.7	0.311255	0.682834	0.825668	0.890821	0.925453	0.945942
	3	0.8	0.393381	0.74129	0.862197	0.91515	0.942675	0.958731
	3	1	0.266244	0.646933	0.803323	0.876113	0.91515	0.938357
	2	1.2	0.382298	0.733647	0.858607	0.913271	0.941585	0.958047
	2	1.5	0.259878	0.636675	0.797724	0.872988	0.913271	0.93715
0.10	2	2	0.130851	0.485785	0.690372	0.797724	0.858607	0.895954
	4	0.7	0.311255	0.682834	0.825668	0.890821	0.925453	0.945943
	4	0.8	0.237448	0.621366	0.785786	0.863824	0.90617	0.931543
	3	1	0.266244	0.646933	0.803323	0.876113	0.91515	0.938357
	3	1.2	0.173371	0.555527	0.74129	0.833312	0.884266	0.91515
	2	1.5	0.259878	0.636675	0.797724	0.872988	0.913271	0.93715
0.05	2	2	0.130851	0.485785	0.690372	0.797724	0.858607	0.895954
	5	0.7	0.196051	0.580088	0.757173	0.843754	0.891512	0.920428
	5	0.8	0.134944	0.508837	0.706051	0.807392	0.864759	0.900052
	4	1	0.13115	0.504563	0.70352	0.805896	0.863824	0.899435
	3	1.2	0.173371	0.555527	0.74129	0.833312	0.884266	0.91515
	3	1.5	0.085459	0.4308	0.646933	0.764772	0.833312	0.876113
0.01	2	2	0.130851	0.485785	0.690372	0.797724	0.858607	0.895954

Table 10: Probability of acceptance for the two- stage group sampling plan with g_1 = 2 & g_2 = 1when the life time of the item follows Weibull distribution

β	r	a	μ/μ₀ 2	4	6	8	10	12
	2	0.7	0.407235	0.718104	0.898436	0.929812	0.929812	0.948663
	2	0.8	0.342799	0.805996	0.87433	0.912274	0.912274	0.93539
	2	1.0	0.242323	0.735603	0.823438	0.87433	0.87433	0.90619
	2	1.2	0.171569	0.666468	0.770813	0.833818	0.833818	0.87433
	2	1.5	0.103044	0.569271	0.692096	0.770813	0.770813	0.823438
0.25	2	2.0	0.045075	0.431099	0.569271	0.666468	0.666468	0.735603
	3	0.7	0.194249	0.534472	0.807572	0.862769	0.862769	0.897418
	2	0.8	0.342799	0.805996	0.87433	0.912274	0.912274	0.93539
	2	1.0	0.242323	0.735603	0.823438	0.87433	0.87433	0.90619
	2	1.2	0.171569	0.666468	0.770813	0.833818	0.833818	0.87433
	2	1.5	0.103044	0.569271	0.692096	0.770813	0.770813	0.823438
0.10	2	2.0	0.045075	0.431099	0.569271	0.666468	0.666468	0.735603
	3	0.7	0.194249	0.534472	0.807572	0.862769	0.862769	0.897418
	3	0.8	0.142305	0.659301	0.766989	0.831564	0.831564	0.872897
	2	1.0	0.242323	0.735603	0.823438	0.87433	0.87433	0.90619
	2	1.2	0.171569	0.666468	0.770813	0.833818	0.833818	0.87433
	2	1.5	0.103044	0.569271	0.692096	0.770813	0.770813	0.823438
0.05	2	2.0	0.045075	0.431099	0.569271	0.666468	0.666468	0.735603
	4	0.7	0.080881	0.377624	0.709312	0.786607	0.786607	0.837224
	3	0.8	0.142305	0.659301	0.766989	0.831564	0.831564	0.872897
	3	1.0	0.074966	0.558206	0.685833	0.766989	0.766989	0.820949
	3	1.2	0.038692	0.46747	0.607615	0.701922	0.701922	0.766989
	2	1.5	0.103044	0.569271	0.692096	0.770813	0.770813	0.823438
0.01	2	2.0	0.045075	0.431099	0.569271	0.666468	0.666468	0.735603

Table 11: Probability of acceptance for the two- stage group sampling plan with g_1 = 2 & g_2 = 1when the life time of the item follows Rayleigh distribution

β	r	a	μ/μ₀ 2	4	6	8	10	12
	3	0.7	0.290651	0.557086	0.709435	0.797119	0.851004	0.886186
	3	0.8	0.249254	0.502932	0.662304	0.758924	0.820244	0.861168
	2	1.0	0.402805	0.621878	0.748915	0.823248	0.869434	0.899828
	2	1.2	0.354099	0.561012	0.694225	0.777992	0.832459	0.869434
	2	1.5	0.29653	0.488276	0.621878	0.714152	0.777992	0.823248
0.25	2	2.0	0.213904	0.402805	0.526192	0.621878	0.694225	0.748915
	4	0.7	0.149927	0.402086	0.58086	0.695102	0.769802	0.820625
	3	0.8	0.249254	0.502932	0.662304	0.758924	0.820244	0.861168
	3	1.0	0.190469	0.413113	0.576623	0.68554	0.758924	0.80996
	3	1.2	0.150938	0.34394	0.502932	0.617969	0.699802	0.758924
	2	1.5	0.29653	0.488276	0.621878	0.714152	0.777992	0.823248
0.1	2	2.0	0.213904	0.402805	0.526192	0.621878	0.694225	0.748915
	4	0.7	0.149927	0.402086	0.58086	0.695103	0.769802	0.820625
0.05	4	0.8	0.118576	0.344409	0.522943	0.644271	0.726717	0.784309

	4	1.0	0.07847	0.255721	0.423652	0.551201	0.644271	0.712577
	3	1.2	0.150938	0.34394	0.502932	0.617969	0.699802	0.758924
	3	1.5	0.109141	0.26868	0.413113	0.529185	0.617969	0.68554
	2	2.0	0.213904	0.402805	0.526192	0.621878	0.694225	0.748915
0.01	6	0.7	0.031904	0.187327	0.36198	0.500734	0.603599	0.67958
	5	0.8	0.051579	0.225406	0.40086	0.535048	0.632751	0.704183
	5	1.0	0.029053	0.149491	0.299842	0.431083	0.535048	0.615617
	4	1.2	0.054785	0.193684	0.344409	0.470606	0.568829	0.644271
	3	1.5	0.109141	0.26868	0.413113	0.529185	0.617969	0.68554
	3	2.0	0.059203	0.190469	0.306914	0.413113	0.502932	0.576623

Table 12: Probability of acceptance for the two- stage group sampling plan with $g_1 = 2$ & $g_2 = 1$when the life time of the item follows inverse Rayleigh distribution

β	r	a	μ/μ₀					
			2	4	6	8	10	12
0.25	3	0.7	0.922393	0.999999	1.000000	1.000000	1.000000	1.000000
	2	0.8	0.886509	0.999999	1.000000	1.000000	1.000000	1.000000
	2	1.0	0.647706	0.999701	1.000000	1.000000	1.000000	1.000000
	2	1.2	0.422084	0.993589	0.999999	1.000000	1.000000	1.000000
	2	1.5	0.214967	0.929811	0.999701	1.000000	1.000000	1.000000
	2	2.0	0.079379	0.647706	0.97678	0.999701	0.999999	1.000000
0.10	3	0.7	0.922393	0.999999	1.000000	1.000000	1.000000	1.000000
	3	0.8	0.787307	0.999998	1.000000	1.000000	1.000000	1.000000
	2	1.0	0.647706	0.999701	1.000000	1.000000	1.000000	1.000000
	2	1.2	0.422084	0.993589	0.999999	1.000000	1.000000	1.000000
	2	1.5	0.214967	0.929811	0.999701	1.000000	1.000000	1.000000
	2	2.0	0.079379	0.647706	0.976780	0.999701	0.999999	1.000000
0.05	4	0.7	0.874838	0.999999	1.000000	1.000000	1.000000	1.000000
	3	0.8	0.787307	0.999998	1.000000	1.000000	1.000000	1.000000
	3	1.0	0.444211	0.999333	1.000000	1.000000	1.000000	1.000000
	2	1.2	0.422084	0.993589	0.999999	1.000000	1.000000	1.000000
	2	1.5	0.214967	0.929811	0.999701	1.000000	1.000000	1.000000
	2	2.0	0.079379	0.647706	0.976780	0.999701	0.999999	1.000000
0.01	5	0.7	0.822140	0.999999	1.000000	1.000000	1.000000	1.000000
	4	0.8	0.681882	0.999996	1.000000	1.000000	1.000000	1.000000
	3	1.0	0.444211	0.999333	1.000000	1.000000	1.000000	1.000000
	3	1.2	0.207138	0.986144	0.999998	1.000000	1.000000	1.000000
	3	1.5	0.059764	0.862768	0.999333	1.000000	1.000000	1.000000
	2	2.0	0.079379	0.647706	0.97678	0.999701	0.999999	1.000000

7. EXAMPLES

Based on the consumer's risk values and the test termination ratios, the minimum number of testers is determined and hence the minimum sample size is obtained. Suppose that the experimenter is interested in establishing that the true unknown average life is at least 1000 hours with β = 0.25. It is desired to stop the experiment at t = 628 hours. It is assumed that c_1 = 0 and c_2 = 2. Following are the results obtained when the lifetime of the test items follows the Generalized Exponential distribution, Marshall – Olkin extended Lomax distribution, Marshall–Olkin extended exponential distribution, Weibull Distribution, Inverse Rayleigh Distribution respectively.

7.1 Generalized Exponential Distribution: Let the distribution followed be Generalized Exponential**,** it is assumed that g_1 = 2, g_2 = 1, c_1 = 0,c_2 = 2 and β = 0.25. From Table 1 the minimum number of testers required is r = 4.Thus we will draw a first sample of size n_1 = 8 items and allocate 4 items to each of 2 groups to put on tester for 700 hours. ($n_1 = rg_1$), if no failure occur during 700 hours we accept the lot. The test is terminated and the lot is rejected if more than 2 failures occurs otherwise if 1 or 2 failures occurs, then we move on to the second stage, where the second sample of size n_2 = 4 ($n_2 = rg_2$) is chosen and tested. For the above conditions the probability of acceptance from Table 7 is 0.99928587 when the ratio of the unknown average life is 12.

7.2 Marshall – Lomax extended exponential distribution: Let the distribution followed be Marshall – Olkin Extended Lomax**,** it is assumed that g_1 = 2, g_2 = 1, c_1 = 0,c_2 = 2 and β = 0.25. From Table 2 the minimum number of testers required is r = 3.Thus we will draw a first sample of size n_1 = 6 items and allocate 3 items to each of 2 groups to put on tester for 700 hours. ($n_1 = rg_1$), if no failure occur during 700 hours we accept the lot. The test is terminated and the lot is rejected if more than 2 failures occurs otherwise if 1 or 2 failures occurs, then we move on to the second stage, where the second sample of size n_2 = 3 ($n_2 = rg_2$) is chosen and tested. For the above conditions the probability of acceptance from Table 8 is 0.948733903 when the ratio of the unknown average life is 12.

7.3 Marshall – Olkin Extended Exponential Distribution : Let the distribution followed be Marshall – Olkin Extended Exponential , it is assumed that g_1 = 2, g_2 = 1, c_1 = 0,c_2 = 2 and β = 0.25. From Table 3 the minimum number of testers required is r = 2.Thus we will draw a first sample of size n_1 = 4 items and allocate 2 items to each of 2 groups to put on tester for 700 hours. ($n_1 = rg_1$), if no failure occur during 700 hours we accept the lot. The test is terminated and the lot is rejected if more than 2 failures occurs otherwise if 1 or 2 failures occurs, then we move on to the second stage, where the second sample of size n_2 = 2 ($n_2 = rg_2$) is chosen and tested. For the above conditions the probability of acceptance from Table 9 is 0.967685289 when the ratio of the unknown average life is 12.

***7.4 Weibull Distribution**:* Let the distribution followed be Weibull, **,** it is assumed that g_1 = 2, g_2 = 1, c_1 = 0,c_2 = 2 and β = 0.25. From Table 4 the minimum number of testers required is r = 2.Thus we will draw a first sample of size n_1 = 4 items and allocate 2 items to each of 2 groups to put on tester for 700 hours. ($n_1 = rg_1$), if no failure occur during 700 hours we accept the lot. The test is terminated and the lot is rejected if more than 2 failures occurs otherwise if 1 or 2 failures occurs, then we move on to the second stage, where the second sample of size n_2 = 2 ($n_2 = rg_2$) is chosen and tested. For the above conditions the probability of acceptance from Table 10 is 0.948662687when the ratio of the unknown average life is 12.

7.5 Generalised *Rayleigh Distribution:* Let the distribution followed be Rayleigh**, ,** it is assumed that g_1 = 2, g_2 = 1, c_1 = 0,c_2 = 2 and β = 0.25. From Table 5 the minimum number of testers required is r = 3.Thus we will draw a first sample of size n_1 = 6 items and allocate 3 items to each of 2 groups to put on tester for 700 hours. ($n_1 = rg_1$), if no failure occur during 700 hours we accept the lot. The test is terminated and the lot is rejected if more than 2 failures occurs otherwise if 1 or 2 failures occurs, then we move on to the second stage, where the second sample of size n_2 = 3 ($n_2 = rg_2$) is chosen and tested. For the above conditions the probability of acceptance from Table 11 is 0.886185967 when the ratio of the unknown average life is 12.

7.6 Inverse-Rayleigh Distribution: Let the distribution followed be Inverse-Rayleigh**, ,** it is assumed that g_1 = 2, g_2 = 1, c_1 = 0,c_2 = 2 and β = 0.25. From Table 6 the minimum number of testers required is r = 3.Thus we will draw a first sample of size n_1 = 6 items and allocate 3 items to each of 2 groups to put on tester for 700 hours. ($n_1 = rg_1$), if no failure occur during 700 hours we accept the lot. The test is terminated and the lot is rejected if more than 2 failures occurs otherwise if 1 or 2 failures occurs, then we move on to the second stage, where the second sample of size n_2 = 3 ($n_2 = rg_2$) is chosen and tested. For the above conditions the probability of acceptance from Table 12 is 1.000000 when the ratio of the unknown average life is 12.

8. CONCLUSIONS

In this paper, designing a two stage group sampling plan for the truncated life test is presented. The minimum number of testers and the probability of acceptance are calculated, when the consumer's risk β and other plan parameters are specified, assuming that the lifetime of an item follows different distributions. When all the above tables (Table 7 to Table 12) are compared, considering both the factors. i.e. minimum number of testers and the probability of acceptance, the Inverse Rayleigh distribution is comparatively the best. It can also be observed that the minimum number of testers required decreases as the test termination time multiplier increases in all the above cited distributions and thus it is concluded that the tables provided can be used conveniently in practical situations when a multiple number of items at a time are adopted for a life test to save the cost and time of the experiment.

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